A SURVEY OF THE DEVELOPMENT OF THE SCIENCE OF HEAT AND MASS TRANSFER IN THE SOVIET UNION

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The science of heat and mass transfer is a broad discipline involving continuum hydrodynamics, thermodynamics, thermophysics, the molecular physics of disperse systems, and chemical kinetics. It has numerous practical applications in power engineering, and in the chemical, food, and light industries, as well as in the manufacture of construction materials. Heat and mass transfer problems are particularly important in jet-engine and rocket design, in nuclear engineering, and in a number of other areas of modern technology.

The science of heat and mass transfer makes wide use of the apparatus of the thermodynamics of irreversible processes, which extends classical thermodynamics to nonequilibrium situations.

The history of the science of heat and mass transfer is a very long one. The theory of heat transfer is part of the general theory of heat, the foundations of which were laid by the great Russian scientist M. V. Lomonosov. Lomonosov's kinetic theory of heat contained all the basic elements of the modern theory, namely: the law of conservation of mass and energy, and the idea of heat as a form of motion associated with the motion of the elementary particles of a body (atoms, molecules).

A contemporary of Lomonosov, G. V. Rikhman, was the first to experiment with the cooling of bodies and with the evaporation of water.

The thermometric studies begun by Lomonosov and Rikhman were pursued without interruption in Russian scientific institutions and always ensured a leading role for Russian science in that area of technology.

A brilliant exposition of the theory of heat conduction can be found in the lectures of A. G. Stoletov (1881–1882). These lectures are remarkable in that they represent the first full combined treatment of the theory of heat conduction and thermodynamics. Thus, Stoletov must be regarded as the originator of thermophysics.

In more recent times the science of heat and mass transfer has grown enormously. Because of its generality, a detailed and all-inclusive account of its development would be impossible. We will therefore concentrate on three areas of particular importance to heat-engineering and chemical-engineering processes.

At the same time, however, it will be necessary at least to outline the basic directions of development of heat and mass transfer.

1. DEVELOPMENT OF THE THEORY OF HEAT CONDUCTION

Mathematical results. A broad class of phenomena—transport of heat, mass, neutrons, etc.—is described in a certain approximation by parabolic equations (or systems of such equations) of the heat-conduction type. In our review, which makes no claims to completeness, we will note the most important trends developed by Societ scientists.

In 1935 A. I. Tikhonov obtained a fundamental result concerning the conditions of existence and uniqueness of solutions of the equation of heat conduction on an infinite spatial interval. He showed that the Cauchy problem is correct in a class of functions growing at infinity not more rapidly than $Ce^{c_1x^2}$; at a more rapid rate of growth the solution is not unique.

S. L. Sobolev developed the theory of partial differential equations, including equations of the parabolic type, by introducing the concept of generalized solutions. A further development was the application of methods from the theory of generalized functions, as a result of which fairly complete descriptions of the classes of uniqueness and correctness from a number of problems were obtained (the work of I. M. Gel'fand and co-workers).

A number of important results relating to the properties of solutions and to the solvability of parabolic equations and systems (for various generalizations of the concept of parabolicity) were obtained by I. G. Petrovskii and his school. O. A. Oleinik, O. A. Ladyzhenskaya and others developed effective methods of investigating quasi-linear equations of the parabolic type. A great contribution to the theory of elliptic and parabolic equations was made by M. I. Vishik using functional methods. He established the conditions of solvability and the properties of the solutions of a broad range of problems, investigating, in particular, their boundary behavior.

Work on the theory of stochastic processes leading to equations of the parabolic or elliptic type also deserves notice, since the properties of the stochastic process provide a considerable amount of information about the properties of these solutions A. N. Kolmogorov, E. B. Dynkin, M. I. Freidlin, and others).

Development of methods of solution. Over a period of years A. V. Luikov and his school have extensively introduced the method of integral transforms for solving various linear problems of the theory of heat and mass transfer. This method has a number of advantages as compared with other methods for solving the boundary-value problems of transport theory.

In particular, the mathematical operations with the transform function are often much simpler than with the original function, owing to the simplification of the equations. For example, the differentiation of the starting function is replaced by a multiplication, using the Laplace transformation. In some cases this makes it possible to reduce a boundary-value problem for a partial differential equation with two independent variables to an ordinary differential equation.

Secondly, various integral transformations can be used to transform a function of one variable (say, time) into the corresponding transform function of another variable (for example, frequency). The transform function may exhibit very simple analytical properties with respect to the new variable and may be more amenable to physical investigation (frequency characteristics of the system, etc.).

Finally, the extensive tables of integral transforms are very useful, especially for engineers who find it necessary to solve problems in the theory of energy and mass transfer.

In general, the integral transformation transforms a given function $f(z)$ with variable z into another function g(w) with variable w, i.e.,

$$g(w) = \int_{C_1} K_1(w,z) f(z)\, dz, \tag{1}$$

where the function K_1 of the variables w and z (generally speaking, complex) is called the kernel of the transformation, and C_1 is the contour of integration.

On the other hand, Eq. (1) is a linear integral equation of the function for $f(z)$, if the function g(w) is given and the kernel K_1 and the contour C_1 are determined. The solution of this integral equation can be formally written as

$$f(z) = \int_{C_2} K_2(z,\ w)\, g(w)\, dw. \tag{2}$$

The latter relation is the inverse integral transformation of (1) and makes it possible to return to the inverse transform $f(z)$ after all the necessary operations have been performed on the transform g(w).

A well-known example illustrating the general principle is the so-called unilateral Laplace transformation. Among other integral transforms frequently employed in transport theory, we note the Fourier cosine and sine transforms, the Fourier exponential transform, the Mellin and Hankel transforms, etc.

The choice of a suitable integral transform, i.e., the choice of the kernel $K_1(w, z)$ and the contour of integration in Eq. (1), depends on the type of equation and the boundary conditions describing the transfer process. Luikov examined this most important practical question in a large number of examples.

Particularly effective, as compared with the ordinary method of eigenfunctions for solving inhomogeneous bondary-value problems, is the method of finite integral transforms developed by G. A. Grinberg and used by him and by other authors to solve various problems of mathematical physics.

The operational calculus and the theory of integral transforms were developed by V. P. Ditkin and coworkers. The introduction of the method of integral transforms led to the appearance of numerous solutions of specific problems in the theory of heat and mass transfer in various branches of science and technology. The extensive work of A. P. Prudnikov deserves particular mention.

Approximate and numerical methods are particularly important in connection with the solution of nonstationary, multidimensional, and nonlinear problems in modern theory of heat and mass transfer.

The theoretical basis of these approximate methods is functional analysis. The general theory of approximate methods has been developed for a broad class of functional equations by L. V. Kantorovich. The same author has developed a functional variant of Newton's method, a very powerful tool both for investigating and for actually solving nonlinear problems.

The method of moments, especially in the modified form of Galerkin's method, is very effective for solving boundary-value problems in heat and mass transfer. Convergence was first demonstrated for differential equations by G. I. Petrov and in the more general case by M. V. Keldysh.

The method of averaging functional corrections developed by Yu. D. Sokolov is useful for solving certain nonlinear problems of diffusion and heat conduction.

Advances in computer technology have led to important progress in numerical methods of solving the problems of methematical physics, including parabolic problems of transfer theory.

The development of network methods has been associated, above all, with the use of implicit difference schemes, including the multidimensional schemes developed by A. A. Samarskii, N. N. Yanenko, E. G. D'yakonov, etc. In particular, for partial differential equations with variable coefficients D'yakonov and Bakhvalov constructed schemes involving no more computation than those for equations with constant coefficients.

Yanenko and Samarskii developed the theory of economic difference schemes for multidimensional parabolic equations in an arbitrary domain.

Samarskii and Tikhonov constructed a theory of homogeneous difference schemes, extending the elementary balance method (A. P. Vanichev) to higher-accuracy approximations.

In all applications of implicit difference schemes the principal problem—solution of the implicit network equations—is usually handled either by suppressing the error components in an iteration process (A. A. Abramov) or by the pivotal method. This remarkable method was devised by M. V. Keldysh, I. M. Gel'fand, and O. K. Lokutsievskii. Factorization methods have been applied to neutron transport problems by G. I. Marchuk.

It should be noted that numerical methods of solving problems of the parabolic and elliptic types (i.e., stationary transfer problem) developed in parallel, since methods of solving one necessarily include the use of the properties of equations of the other type and conversely.

Special problems of the theory of heat conduction. The process of heat transfer in the presence of phase transitions in the system is usually described by the Stefan problem. An approximate method of solving this problem was proposed by L. S. Leibenzon. L. I. Rubinshtein investigated the existence and uniqueness of the Stefan problem. V. G. Melamed developed an effective method of solving the Stefan problem by reducing it to the solution of an infinite denumerable system of ordinary differential equations. This method is highly suitable for computerization.

Effective difference methods (including some for multidimensional problems) have been developed by A. A. Samarskii and co-workers (these methods are based on the extension of the method of homogeneous difference schemes to the case in which the coefficients of the equations have singularities associated with a phase transition) as well as by O. A. Oleinik and B. M. Budak and co-workers.

In conclusion it should be stressed that the physical formulation of the Stefan problem itself requires refinement.

A very important class of problems involves heat conduction with temperature-dependent sources. These problems are a consequence of the work of N. N. Semenov and his school on chemical kinetics and, in particular on the theory of chain reactions. The thermal theory of ignition was developed by Ya. B. Zel'dovich, D. A. Frank-Kamenetskii, and many others.

Thermal explosion may occur when heat release increases in the system exponentially with the temperature, while the rate of heat removal remains almost linear. This leads to continuous autoacceleration of the chemical reaction, ending in explosion.

A detailed review of work on the stationary and nonstationary theory of thermal explosion can be found in Frank-Kamenetskii's monograph. Among later work we note that of Zel'dovich, G. I. Barenblatt, and A. F. Dubovitskii and co-workers.

Obviously, the mathematical theory of thermal explosion is far from complete.

The next important class of heat-conduction problems has been investigated by Zel'dovich and A. S. Kompaneits.

These authors have investigated various boundary-value problems for an equation of the heat conduction type with thermal conductivity depending exponentially on temperature. The most important characteristic of the heat-propagation process described by such an equation, differentiating it from the ordinary process of linear heat conduction, is the finiteness of the heat-propagation velocity. In the linear theory the asymptotic character of the temperature decrease at large distances from the source is associated with the finiteness of the thermal conductivity as the temperature tends to zero. In the case considered, owing to the vanishing of the thermal conductivity far from the source (as T → 0), the heat cannot penetrate to an infinite distance, but is propagated at a finite velocity such that at any time there is a distinct boundary (wave front) separating the "cold" and "hot" regions.

Later on, the same effect will be considered from another standpoint.

Numerical methods of solving thermal wave problems have been developed by A. A. Samarskii and co-workers.

Finally, we will briefly consider some very important work on certain fundamental problems of transport theory associated with refinement of the starting equations. As already noted, parabolic transport equations (for example, the ordinary diffusion equation), give an idealized description of a transport proceeding at infinite velocity, in the sense that any interaction is propagated over an infinitely large distance from the source in an arbitrarily small interval of time.

A refined diffusion equation in which this paradox is eliminated was derived by V. A. Fok.

Based on the generalization of the methods of the thermodynamics of irreversible processes A. V. Luikov has shown it is necessary to generalize the ordinary relation linking a flux with the corresponding thermodynamic force. For example, for mass flow the new relation has the form:

$$\mathbf{j} = -D\nabla c + \tau_0 \frac{\partial \mathbf{j}}{\partial \tau}, \tag{3}$$

and, consequently, the mass-transfer process is described not by the diffusion equation but by an equation of the telegraph type with a sharply defined leading wave front propagating at finite velocity.

From general relation (3) it is easy to establish the limits of applicability for the approximation of "transport at infinite velocity." Obviously, this occurs when the mean free path tends to zero and the velocity to infinity, but so that the diffusion coefficient $D \sim vl$ remains constant. In this case the ordinary parabolic equations are a good approximation to reality and to the exact kinetic integrodifferential equations.

However, in some cases (moisture transport in capillary-porous bodies, neutron diffusion) either the velocity may be small (in capillary-porous body) or the free path may be large (neutron transport). In these important practical cases the finiteness of the propagation velocity can no longer be neglected.

2. DEVELOPMENT OF EXPERIMENTAL STUDIES OF THERMOPHYSICAL CHARACTERISTICS

The thermophysical characteristics (thermal conductivity, thermal diffusivity, and specific heat) of poor thermal conductors are of great importance not only for heat-engineering calculations but also for formulating physical experiments in a number of branches of modern thermophysics.

For a long time thermal conductivity was the only thermophysical characteristic. Russian scientists paid great attention to experimental methods of determining the thermal conductivity of both poor and good conductors.

They were the first to develop experimental methods of determining the thermophysical coefficients of various materials, F. F. Petrushevskii, chairman of the Physical Division of the Physicochemical Society, and his scientists devised unique methods of determining thermal conductivity.

Professor R. E. Lents (the son of Academician E. Kh. Lents) was the first to demonstrate experimentally the constancy of the ratio of the thermal and electrical conductivities of metals. Lents overcame great experimental difficulties that had defeated Wiedemann—who suggested the constancy of the ratio—and other West European scientists.

Most experimental determinations of thermal conductivity have been based on the stationary heat flow method. As a rule, efforts were made to create a one-dimensional temperature field in the sample by prolonged heating with a constant heat flux. The thermal conductivity was determined as the ratio of the specific heat flow to the temperature difference in the investigated sample. Special devices (protective rings, auxiliary heaters, etc.) were introduced to prevent leakages through the lateral surfaces of the relatively thin specimens.

In spite of the apparent simplicity of this method it has the disadvantage of requiring a lengthy experiment, since a relatively long time is required to establish a stationary temperature field. There are serious difficulties associated with creation of a simultaneous temperature field and the elimination of heat losses.

The stationary heat-flow method was developed in various modifications for both plastic and granular materials, which are poor heat conductors. Important improvements were introduced by workers at the Physicotechnical Laboratory of the Dzerzhinskii All-Union Thermal Engineering Institute (VTI) (1925–1940). This laboratory (D. L. Timrot, etc.) developed a number of instruments for determining the thermal conductivities of thermal insulators, powered materials, etc. These instruments were widely employed in factory laboratories and in industry. The introduction of these methods was assisted by engineering studies at the "Teploproekt" Combine.

Apart from the above-mentioned disadvantages, the method does not permit determination of the thermal diffusivity and hence the specific heat; it also introduces serious errors in determining the thermal conductivities of moist materials. Under the influence of the temperature difference there is a redistribution of mositure and, moreover, the liquid moisture is partially evaporated. Different results are obtained depending on the duration of the experiment.

All this made it necessary to develop new methods that would make it possible to determine both the thermal conductivity and the thermal diffusivity in a single, relatively short experiment. A start in this direction was made by G. M. Kondrat'ev at the Leningrad branch of the VTI.

Regular regime methods were developed on the basis of the laws of a nonstationary temperature field for a body cooled in a medium at constant temperature. These were based on the following principle: after a certain interval the temperature at any point of a homogeneous isotropic solid varies with time according to a simple exponential law as it is heated or cooled.

Kondrat'ev and his colleagues made a detailed investigation of the laws of cooling (heating) of solids in the regular regime stage. Starting in 1930, these methods were widely introduced for measuring thermophysical characteristics and retain an important position among modern experimental methods.

Almost simultaneously, in the VTI Drying Laboratory a method was proposed for determining thermophysical characteristics, on the basis of the laws of the nonstationary temperature field, associated with a body heated in a medium whose temperature is a linear function of time. This method of the regular regime of the second kind and various modifications were widely introduced.

Somewhat later (1942–1955) methods of determining the thermophysical characteristics of granular materials were developed, on the basis of the laws of the nonstationary temperature field for a body cooled in an infinite medium in the Physics Departments of the Moscow Technological Institute of the Food Industry (MTIPP) and the Moscow Institute of Chemical Machine Building (G. I. Krasovskaya, V. L. Shevel'kov, K. L. Sheptunov, M. V. Kulakov and others). These methods, called infinite standard methods, found wide application in the chemical, food, and light industries for determining the thermophysical characteristics of various granular and bulk materials.

Probe methods are also used for determining the thermophysical characteristics of bulk materials and soils. These methods were developed in the Agrophysical Scientific Research Institute (A. F. Chudnovskii and co-workers), in the Thermal Insulation Laboratory of the Ministry of the Petroleum Industry (L. F. Yankelev), in the Drying Laboratory of the All-Union Scientific Research Cinema-Photographic Institute (E. E. Vishnevskii), and in a number of other establishments.

All nonstationary methods possess a number of advantages over a stationary heat-flow methods. They make it possible to determine from one short experiment all the thermophysical characteristics of poor heat conductors. Since the experiment is short, there is almost no redistribution of moisture of evaporation in the test materials. Hence, the data obtained correspond to the true values of the characteristics.

Such methods are widely used in investigating semiconductors. In this connection, one should mention the work being done at the Ioffe Physicotechnical Institute and by the Molecular Physics Department of Moscow State University (Department Head A. S. Predvoditelev).

During the last 10–15 years extensive experimental material on the thermophysical characteristics of insulating, powdered, and granular materials has been accumulated, and now the problem arises of obtaining semiempirical design formulas for predicting the thermophysical characteristics of a composite material on the basis of the characteristics of its components.

This problem is being investigated at the Leningrad Institute of Precision Mechanics and Optics (G. N. Dul'nev and co-workers) and also at the Institute of Heat and Mass Transfer AS BSSR. *

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*For a report on the work being done at the Institute of Heat and Mass Transfer AS BSSR see the article by G. M. Volokhov et al. in this issue.

An important practical problem is that of determining the thermophysical characteristics of capillary-porous bodies with various fillers, both at high temperatures (1500–3000° C) and in the region of low (helium) temperatures. The general trend is to determine the thermophysical characteristics on the basis of the laws of a nonstationary temperature field, in the presence of phase transitions, chemical reactions, and variation in the structural-mechanical and physicochemical properties of the material when heated to high temperatures. Consequently, the thermophysical characteristics are effective values.

Naturally, this approach is dictacted by the urgent need to determine the thermophysical characteristics of a series of construction materials. Obviously, the true thermophysical characteristics will eventually be determined independently of the heats of phase transitions and chemical reactions. To solve this problem it is necessary to investigate the laws of nonstationary temperature and concentration fields, which are determined by the solutions of a system of interrelated differential-integral equations of heat and mass transfer.

3. DEVELOPMENT OF FLUIDIZED-BED RESEARCH IN THE SOVIET UNION

In the fluidized state dispersed materials acquire remarkable properties of fluidity and high and easily regulated effective thermal diffusivity. The heat-transfer coefficient of a gas-fluidized bed can be brought almost to the values obtainable with liquids. In the Soviet Union research on fluidization began in the thirties. For the most part, this work was concerned with the gasification and combustion of fine-grained fuel. Results were published by M. K. Pis'men (1938) and B. K. Klimov (1939). Other studies of the combustion of fuel in a fluidized (or spouting) bed were made at the VTI by V. P. Romadin and A. Gorbanenko.

During and after the war most fluidization research in the Soviet Union continued to be directly concerned with individual technical processes such as fuel gasification and combustion and drying.

Work on the gasification of pulverized coal in various devices was published by N. V. Shishakov, and V. G. Kashirskii, work on the gasification of coal and milled peat by P. Kh. Kurinov and others. Kh. I. Kolodtsev and co-workers studied gasification in a fluidized bed of graphite. A brief discussion of proposed fluidized-bed dryers was published in M. Yu. Lur'e's textbook on drying technology. N. I. Syromyatnikov obtained patents for fluidized-bed furnaces, including one with a vibrating distributor, and published, together with N. I. Reshetin, data on the combustion of coke fines in a fluidized bed. N. A. Semenenko and L. N. Sidel'kovskii reported on the combustion of culm, coke fines, and brown coal in a fluidized bed. N. V. Pavlovich described semi-industrial experiments with a two-stage fluidized-bed furnace burning Ukrainian brown coal.

However, even in the forties the fundamental hydrodynamic and transfer processes in the fluidized bed were already being investigated.

Thus, I. M. Fedorov began to study the heat transfer between gases and solids in fluidized beds, A. A. Voitekhovskii, A. P. Zinov'ev, and D. I. Orochko analyzed the hydrodynamics of powder fluidization, and S. M. Obryadchikov and B. K. Marushkin made model studies of the fluidized bed.

During the fifties the volume of basic research increased sharply. N. I. Smirnov and co-workers published a series of papers on fluidized-bed hydrodynamics, I. G. Martyushin investigated the motion of gases and solids in fluidized beds, the effect of temperature on the fluidization process, and the hydromechanical design of fluidization apparatus.

The motion of a gas in a fluidized bed was studied by K. P. Lavrovskii and A. L. Rozental.

D. M. Mints and his colleagues published their work on fluidization in an ascending liquid flow and particle settlement, in connection with the design of new high-speed filters. N. I. Syromyatnikov developed the statistical theory of formation of fluidized beds. P. G. Romankov and co-workers investigated the aerodynamics of fluidization in narrow channels, and G. P. Sechenov and V. S. Al'tshuler made the first systematic study of fluidization at elevated pressures.

Problems of fluidization aerodynamics were examined at the Institute of Heat and Mass Transfer and a new approach to the design of efficient gas distributors was proposed.

M. S. Sharlovskaya investigated bed hydrodynamics in the region of transition to the fluidized state. I. M. Razumov and L. I. Larionova published work on the entrainment of solids from a fluidized bed. Particle mixing was investigated by A. K. Bondareva and O. M. Todes, and "diffusion" of solids from the bed through an opening in the wall by E. F. Kurgaev.

O. M. Todes and co-workers, generalizing the results of a series of experimental studies of fluidized-bed hydrodynamics, were the first to propose convenient interpolation formulas for calculating the expansion of relatively homogeneous beds, formulas which took the place of the previous laborious grapho-analytical methods of Richardson and Zaki.

During this period the rapid accumulation of information made it possible to establish many laws of heat and mass transfer in fluidized beds.

N. A. Shakhova experimentally investigated the heat transfer in an inhomogeneous fluidized bed by a nonstationary method and was one of the first to arrive at a correct conclusion concerning the reson for the rather low Nu numbers obtained. M. S. Sharlovskaya obtained information about particle heat transfer in the presence of weakly developed fluidization. M. S. Verteshev and A. A. Komarovskii studied mass transfer in a fluidized bed in the process of dissolution of salts. A. P. Baskakov and N. I. Syromyatnikov gave a simplified method of calculating the heating time for the material in a bed. The behavior of complex three-component fluidized systems of the gas-vapor-solids type was investigated for the first time. This research was reported by A. V. Chechetkin in his monograph on high-temperature heat-transfer agents.

In studying the effective thermal conductivity of a fluidized bed, A. K. Bondareva detected a maximum depending on the filtration rate and determined the effect of particle diameter on this maximum.

N. I. Gel'perin, V. Ya. Kruglikov, and V. G. Ainshtein experimentally investigated the heat transfer of individual tubes immersed in a fluidized bed.

On the basis of an analysis of extensive experimental scientists at the Institute of Heat and Mass Transfer AS BSSR first proposed and approximate theory of heat transfer in a homogeneous fluidized bed that accounted for many phenomena previously considered anomalous. A simple interpolation formula for practical calculations of the heat transfer in actual inhomogeneous gas-fluidized beds was also proposed. The analysis made at the Institute of Heat and Mass Transfer was then confirmed by the data of a new experimental investigation published by I. P. Mukhlenov and V. B. Sarkits and others. Mukhlenov and co-workers supplied correlations of their own data. Heat transfer in a fluidized bed was also investigated by N. N. Varygin and I. G. Martyushin who correlated their experimental data by means of dimensional analysis.

D. I. Orochko and T. Kh. Melik-Akhnazarov and co-workers published a series of papers on the generation of so-called stepped counterflow in fluidized-bed apparatus, an effect favorable not only thermally but also for chemical reactions.

During the fifties P. G. Romankov and co-workers, A. N. Planovskii and co-workers, O. M. Todes, K. P. Lavrovskii, A. L. Rozental, G. K. Boreskov, and M. G. Slin'ko, I. I. Ioffe, and A. G. Grigorov all published papers on problems of adsorption and chemical reactions in fluidized beds.

During the same period there was much research into roasting, drying, gasification, and other technical processes in fluidized beds. M. K. Pis'men, G. P. Sechenov, and V. S. Al'tshuler published new work on gasification.

Kh. I. Kolodtsev and B. L. Zharkov investigated the gasification and combustion of coal in a fluidized bed, while V. S. Aliev and co-workers studied the combusion of coke. N. I. Syromyatnikov, N. A. Semenko, L. N. Sidel'kovskii, and N. V. Pavlovich reported new findings on the combustion of natural fuels in fluidized beds.

Considerable success was achieved in applying fluidized-bed techniques to nonferrous metallurgy. The process of obtaining powdered iron from pyrite cinders in a fluidized bed was investigated at the Leningrad Technological Institute.

In the same area P. P. Budnikov and co-workers studied the swelling of volcanic materials and clays in a fluidized bed, E. I. Khodorov and V. M. Kosareva reported on the roasting of cement clinker and V. M. Dement'ev on the design of multizone calcining furnaces for burning lime.

The sixties were characterized by the further development of fluidization research. Many old ideas, for example, in the area of heat treatment of metals, were revised and improved in the light of the knowledge that had been gained.

It is impossible to consider all the numerous publications on individual technical processes that appeared during the sixties. We will mention only some of the basic research in fluidized-bed hydrodynamics and heat transfer that was later found to have important practical applications.

The structure of a fluidized bed is extremely complex. It is not only "microscopically" two-phase (medium-particles) but, in the majority of cases, essentially macroscopically inhomogeneous and anisotropic. At any given moment the particle concentration at different points in the bed is not the same, and the motion of the particles is a combination of a random and a more organized circulatory motion. However, for certain purposes it is possible to employ simplified models, for example, the kinetic model proposed recently by V. G. Levich and V. P. Myasnikov who postulate random motion of the particles. The kinetic theory has made it possible to demonstrate the presence in a fluidized bed of a certain effective surface tension at the bed-gas interface—a discovery which throws new light on the origin of cavities (bubbles) in a fluidized bed. Other causes of bubble formation, depending on the design of the apparatus and the operating conditions, are revealed by other approaches. In this connection the work of I. G. Martyushin and N. A. Shakhova should be mentioned.

I. G. Martyushin published a hydrodynamic theory of fluidization according to which fluidization is the conversion of a viscoelastic medium (such as a granular material) into a medium with only viscous (quasiliquid) properties, when the normal stresses in the bed become zero. A perfectly homogeneous fluidized state develops when the "loose" bed structure becomes stable. Instability corresponds to local imbalances of the body and surface forces in the bed which leads to the temporary formation of internal stresses (normal stresses) and discontinuities—the formation of regions free of solids—that is, bubbles and pockets.

Fluidization of binary polydisperse beds, especially in the transition region, and the limits of existence of a polydisperse fluidized bed studied by N. B. Kondukov and co-workers, while V. N. Petrov examined the fluidization of a bed with particles differing not only in size but also in density.

A monograph by V. S. Al'tshuler and G. P. Sechenov deals with fluidization at above-atmospheric pressures, which is also the subject of papers by I. P. Mukhlenov and co-workers (pressures up to 230 atm) and O. S. Chekhov and others.

The first published data on the effect of magnetic fields on the fluidization of beds of ferromagnetic particles were reported by I. M. Kirko with M. V. Filippov and by Z. I. Nekrasov with V. V. Chekin.

Some aspects of hydrodynamics of high-temperature systems with and without chemical reactions have been investigated at the Institute of Heat and Mass Transfer by A. M. Gulyuk and N. V. Antonishin and at the Steel Institute by M. A. Glinkov and V. V. Belousov. In the case investigated, chemical reaction reduced the fluctuation of the bed.

Soviet research into fluidized-bed hydrodynamics has always aimed at achieving the ability to control the properties of fluidized beds and modify them in favorable directions. For example, there have been no detailed mathematical studies of the motion of an idealized bubble in a so-called "normal fluidized bed with bubbles," such as are to be found in the English literature; instead much has been done to eliminate undesirable "normal" bubbles, reduce their size, and improve interphase contact.

We will now consider the question of heat and mass transfer. Traditionally there are three principal aspects of heat transfer in a fluidized bed: 1) heat transfer between the particles and the medium (or so-called interphase heat transfer); 2) the effective thermal conductivity of the bed, and 3) heat transfer between the bed and its confining surfaces.

The first problem is not considered acute. In fact, in most cases owing to the highly developed particle surface per unit volume of the bed and the low Biot number, it is possible to take the temperature of the medium at the outlet as approximately equal to the local particle temperature and calculate it simply from the heat balance. This has long been known. Relatively recently L. K. Vasanova and N. I. Syromyatnikov, N. I. Gel'perin, V. G. Ainshtein and others showed that the criterial equations of many authors proposed for calculating the interphase heat transfer in fluidized beds were obtained precisely for those conditions under which the kinetic equations degenerate into the balance equations and the complicated form is clearly not justified by the meager content.

However, there may be a need for kinetic calculations of particle-gas heat transfer in connection with thin beds operating under forced conditions. This is confirmed by experiment. Moreover, a clear picture of interphase heat transfer is important in relation to the mechanism of heat transfer between the bed and the walls and the mechanism of high-speed cooling or heating of gas jets in a fluidized bed.

Accordingly, the problem has been analzyed by workers at the Institute of Heat and Mass Transfer AS BSSR and it has been shown that the effective Nu and α of the particles is a fluidized bed may be several orders lower than the minimum theoretical values corresponding to "pure" conduction. The main reason for the difference between the true and effective values is the nonuniform gas distribution in the bed.

Another reason, as originally shown by Z. F. Chukanov and also by B. N. Vetrov and O. M. Todes, is the effective longitudinal heat conduction.

N. I. Syromyatnikov and co-workers and E. A. Kazakova and co-workers confirmed that the height of the active zone of interphase heat transfer is quite small. Syromayatnikov and co-workers and later N. A. Shakova and A. G. Gorelik have used the regular regime method to investigate interphase heat transfer in fluidized beds.

Obviously, any method increasing the homogeneity of the bed structure is useful for intensifying interphase heat transfer, as well as measures for reducing the longitudinal and, in the case of cross flow, the transverse mixing of the solid phase.

The Institute of Heat and Mass Transfer AS BSSR and the Ural Polytechnic Institute have studied heat transfer in relation to different systems of organized motion of the bed and the gas flow at various mixing rates.

Workers in the laboratories of the Institute of Heat and Mass Transfer have also developed new types of fluidized-bed heat-exchangers with interphase heat transfer and in some cases drying: high-temperature heat-exchangers with a circulating packing, rotor dryers, and a dryer with alternate cold-air and hot-gas cycles for oscillatory drying; rotor air heaters, etc.

I. L. Lyuboshits et al. of the same institute have also determined rational oscillation parameters for drying heat-sensitive materials in a fluidized bed.

Another aspect of heat transfer in a fluidized bed is the effective thermal conductivity or the effective thermal diffusivity, which differs by a multiplier (the volume specific heat of the bed).

O. M. Todes and workers at the Institute of Heat and Mass Transfer established long ago the reasons for the very high effective thermal conductivity of a fluidized bed—the intense transfer of heat by the rapidly moving particles, whose volume specific heat exceeds that of the gas by many hundreds (1000) times. Only if the observed high rate of mixing of the solid phase is ignored does the effective thermal diffusivity achieved in a fluidized bed (up to 110 $\mathrm{cm}^2/\mathrm{sec}$ in experiments at the Institute of Heat and Mass Transfer, almost 60 times greater than the thermal diffusivity of silver) seem improbably high.

The situation is less satisfactory with respect to quantitative data and their correlation. Since the effective thermal diffusivity is an analog of the solid-phase mixing coefficient, we encounter all the above-mentioned difficulties associated with the complex nature of the motion of the bed material (combination of random and circulatory motion).

We will not list the work that has been done on the effective thermal conductivity or thermal diffusivity of the fluidized bed. This includes essentially all the studies of solid phase mixing.

The third aspect of heat transfer in fluidized beds is the question of heat exchange with immersed surfaces or boundary walls.

Apart from the already-mentioned approximate theory in which the principal thermal resistance to heat transfer is assumed to be the "contact" resistance of the layer of gas between the wall and the first row of particles, the "packet" theory, originally proposed abroad, has also been developed. With the object of obtaining an empirical equation in criterial form, O. M. Todes used this theory to propose a new criterion N*, better justified in this contex than Nu. A. P. Baskakov has proposed taking into account an additional "contact" thermal resistance at the boundary of the packet, analogous to that employed in approximate theory.

In the sixties a number of experiments were performed on heat transfer between surfaces and a fluidized bed; in some of these the instantaneous and local transfer coefficients were determined; at the same time heat transfer in complex and little-studied cases of fluidization were investigated (at high temperatures, under vacuum, in the presence of vibration, etc.).

For lack of space it is not possible to discuss this work nor the studies of fuel combustion in fluidized beds, and numerous other investigations associated with various technical applications of the fluidized bed: drying, roasting, heterogeneous chemical reactions, and so on.

During the last seven or eight years about a thousand publications on fluidization have appeared in the Soviet Union. Many of these were eventually translated. A number of papers by Soviet scientists active in this field have been published in the International Journal of Heat and Mass Transfer. Much fluidized-bed research has been discussed at conferences and symposia in Moscow, Minsk, and other cities.

In general, as may be judged even from this incomplete review, the Soviet Union does not lag behind other countries in fluidized-bed research.

4. DEVELOPMENT OF RESEARCH ON HEAT AND MASS TRANSFER IN THE DRYING PROCESS

The drying theory is based on the theory of heat and mass transfer, physicochemical mechanics, and partly on the physics of surface phenomena and disperse systems.

The drying of moist materials is a complex process during which changes occur in the physicochemical, biological, and technical properties of the material. Thus, when a moist material is heated, not only is moisture removed but the structural-mechanical and technical properties also undergo changes.

From the standpoint of the action of heat on moist material the drying process represents the interaction between the material and the ambient medium. In most cases the interaction involves the moist material and moist air. Moist air is not only a heat-transfer agent but, at the same time, a moisture absorber (moisture carrier). Accordingly, it is quite natural that the first work on the theory of drying should have been principally devoted to the investigation of moist air.

The lectures of Professor L. K. Ramzin at Moscow Technical College, published in 1918, contained the I-d diagram of moist air. In this diagram he established for the first time a graphoanalytical relation between the enthalpy of moist air and its moisture content. This made it possible to represent the process of evaporation of water from a free surface in the form of thermodynamic figures. Ramiz successfully applied this method and proposed a formula for the drying of moist materials in various dryers.

Ramzin's work is one of the most important and fundamental results in the development of drying theory. His proposed relations—analytic method—made it possible to represent the drying process of moist materials on the I-d diagram in different variants and for different dryers.

This method was used to obtain the technical characteristics of dryers: the heat consumption per kilogram of evaporated moisture, the amount of dry air required to remove the moisture, the initial drying parameters (temperature, relative humidity, and velocity of the air), and the end parameters. This gave dryer design a scientific basis.

A similar diagram, the Mollier I-x diagram, did not appear in the West until much later. Even today the method of desigining dryers given in foreign manuals is much more complicated and less elegant than the Soviet method.

However, it should be mentioned that all the above calculations are valid only for the evaporation of water from a free surface, i.e., for the period of constant drying rate. In those early days the kinetics of the drying process were little known and in most cases the drying of moist materials was assumed to be identical with the process of evaporation from a free surface.

Ramzin pursued his scientific activities within the walls of the Dzerzhinskii All-Union Thermal Engineering Institute (VTI), which was established on the initiative of Lenin.

Starting in 1923, the Drying Laboratory of the Institute became the leading scientific organization in the area of drying theory and practice. Some of the Institute's importance is attributable to the fact that immediately after its foundation it began to publish first a "Byulleten," and then an "Izvestiya."

The Drying Laboratory of the VTI was simultaneously a training center for drying engineers. N. M. Mikhailov, M. Yu. Lur'e, Chief Designer A. P. Voroshilov, and the director of the laboratory K. V. Bondarenko successfully continued the research begun by Ramzin.

By the early thirties the laboratory had developed a number of original and, for that time, quite advanced dryers for a variety of materials. The staff successfully investigated rotary dryers, chamber dryers, tunnel dryers, spray dryers, etc. Moreover, they developed improved methods of dryer design.

However, tests showed that the drying of moist materials is not identical with the process of evaporation of water from a free surface. The drying rate varies with time.

In the absence of a knowledge of the laws of the processes of heat and moisture transfer in moist materials, dryer design using existing methods require a preliminary experiment on small specimens to obtain data on the release of moisture from the materials at different points during the drying process.

Thus, at the beginning of the thirties there were two main trends in drying theory; the study of the thermophysics of the process, i.e., the kinetics and dynamics of the drying process on the basis of the theory of heat transfer and diffusion, and, the study of the physicochemical and technological properties of the drying material. A considerable contribution to the development of this branch of drying theory was made by foreign scientists, in particular, Lewis, Sherwood, and their students.

The development of drying technology received an important impetus from studies of the nature of the moisture bond. These studies were initiated by

P. A. Rebinder and his followers. S. M. Lipatov, Yu. L. Kavkazov, and S. I. Sokolov investigated the drying process and the physicochemical properties of moist materials.

Basic research in this field was conducted at the Institute of Colloidal Chemistry of the Academy of Sciences, at the Technological Institute of Light Industry, and at the Central Scientific Research Institute of the Leather and Shoe Industry (TsNIKP).

Progress was stimulated by the fact that the drying technologists had good communications with leading scientists in the field of physical chemistry of surface systems (P. A. Rebinder, V. A. Kargin, S. M. Lipatov, etc.).

Attention was concentrated on such phenomena as the shrinkage of moist materials during the drying process, changes in the degree of swelling, etc., i.e., purely technical problems. The investigation of these problems paralleled the study of the processes of sorption and desorption of moist materials interacting with moist air. The analysis of the sorption and desorption isotherms and the calculation of the heats of sorption and desorption were at that time pressing problems of the physical chemistry of moist materials. This area of research, known as tensometry, made good use of x-ray structural analysis, calorimetry, and other physicochemical methods. Important progress was made in understanding the way in which colloidal capillary-porous bodies bind moisture and interact with moist air. This research formed the basis of the one of the most important branches of drying theory—the stacks of drying processes.

As already noted, studies of the thermophysics of the drying process began approximately in 1930–1932. Up to that time the kinetics of drying processes had been only partially investigated and was regarded as a problem secondary to purely technical research.

At this point it is necessary to mention the work of the Central Scientific Research Institute of Wood Machining, in particular, the work of B. A. Posnov and E. M. Lyubimov. Lyubimov was the first to attempt to apply the solution of the differential equation of moisture diffusion for a half-space to the drying of wood.

At that time the VTI Drying Laboratory was experimenting with the mechanism of moisture of migration during the drying process. Sherwood's theory of the depression of the evaporation surface was confirmed by direct experiments and the theory was further developed.

In particular, the problem of moisture diffusion inside a material during the drying process was solved for boundary conditions of the third kind. Sherwood himself acknowledged the value of the work being done in the VTI laboratory.

At the same time, the Leningrad branch VTI was determining the moisture diffusion coefficient for various ceramic materials (I. I. Paleev, Ya. M. Miniovich, and A. A. Shumilin). This early thermophysical research was reported in the leading physical and chemical journals.

Studies of the nonstationary moisture content field in drying materials and nonstationary temperature fields in the VTI Drying Laboratory revealed a new effect—the migration of moisture in the direction of heat flow. This important effect was called the thermodiffusion of moisture.

Work on the thermodiffusion of moisture attracted attention abroad. In particular, reports were published in the Proceedings of the Royal Society (1936).

The results obtained made it possible to explain the cracking of ceramic materials heated in an atmosphere saturated with water vapor.

On the basis of the thermodiffusion effect, workers at the TsNIKP Drying Laboratory (N. S. Solov'ev, Yu. L. Kavazov, and others) successfully developed optimum drying regimes for leather with minimum loss of tanning agents.

It should be noted that when it was first introduced the theory of depression of the exploration zone encountered criticism. This is quite understandable. In fact, in science all novel ideas are greeted with some mistrust.

However, in the course of the years the theory has fully proved itself. Now its thermophysical laws are being successfully used not only in drying technology but also in other areas of science.

In 1938 the research being conducted in the drying laboratories of the VTI, TsNIKP, and other institutes was analyzed in a monograph entitled "The kinetics and dynamics of the drying processes of moist materials." This monograph gave a detailed description of the experimental methods used in investigating the kinetics of drying processes.

As distinct from foreign scientists, the Soviet school of thermophysicists and drying technologists based the analysis of the kinetics of the drying process not only on drying-rate curves but also on so-called temperature curves (material temperature versus moisture content).

Thus, even at that time the drying process was being correctly analyzed as a unified interrelated process of heat and mass transfer.

Thus, at the beginning of the forties, drying theory in the Soviet Union was finally established as an independent branch of knowledge in close touch with practice and soundly based on the fundamental principles of physics and physical chemistry.

Though interrupted by the war, once the fighting was over, progress in drying theory continued to be made at an ever-increasing rate.

At the beginning of the forties drying research was being conducted on a broad front in the food industry, chiefly at the MTIPP. This research involved studies of infrared and vacuum drying, as well as ordinary convective drying.

At about that time freeze drying was introduced. This was a process developed principally by food-industry scientists. Progress in this direction was reviewed in the book "Investigation of the Drying Processes of Colloidal Capillary-Porous Materials in the Food Industry." It whould be noted that this research involved close collaboration between thermophysicists and technologists (L. Ya. Auerman, A. S. Ginzburg, etc.).

At the same time, progress was being made in determining the thermophysical characteristics of moist materials. The dependence of these characteristics on the moisture content was linked with the laws governing the nature of the moisture bond.

Moreover, moisture transport in capillary-porous bodies, in elementary capillaries, and in semibounded media was being investigated. The mathematical apparatus of the theory of heat conduction was widely used. In particular, specialists solved nonstationary problems of the theory of heat conduction with application to the heating of moist materials. This research on nonstationary heat conduction was published in "Diffusion and Heat Conduction" (Gizlegprom, 1941) and "Heat Conduction in Nonstationary Processes" (Gosenergoizdat, 1948).

Thus, the powerful mathematical apparatus of the theory of heat conduction and diffusion came to be widely employed in drying theory. At the end of 1940, as a result of extensive research at the MTIPP, the Moscow Power Engineering Institute, and elsewhere, it became clear that the process of heat and moisture transfer during drying cannot be described by separate differential equations of heat conduction and diffusion. Accordingly, an attempt was made to describe the interrelated process of heat and moisture transfer by means of interrelated differential equations of heat and mass transfer.

The starting relation in deriving this system of differential equations is the assumption that the intensity of the moisture source is directly proportional to the local derivative of the moisture content with respect to time (F. M. Polonskaya). At first, this was only a hypothesis relating the differential equation of heat transfer with the differention equation of moisture transfer. However, it was then shown that the hypothesis can be obtained by purely analytical means for one-dimensional problems of heat and moisture transfer. This relation was determined by a single physical factor, namely: the moisture content at any point of a moist body is almost (correct to 0.001%) equal to the liquid content, i.e., the mass of moist air in capillary and porous bodies is negligibly small as compared with the mass of liquid.

At the same time the methods of the theory of similarity or the theory of generalized variables, developed in the Soviet Union by A. A. Gukhman and M. V. Kirpichev, were widely introduced in drying theory. A series of new criteria, named in honor of leading scientists in the field of the physics of moist materials, was obtained from the system of differential equations.

At the end of 1940 the analytical part of drying theory constituted a certain generalization of classical heat conduction and diffusion theory. In fact, analytical drying theory was the first attempt to create a thermodynamics of irreversible process of heat and mass transfer.

In the late forties and early fifties attempts were made to establish the priority of Soviet scientists in various areas of science and technology. As a result of this historical research it was confirmed that the basic laws of the mechanism of moisture migration in moist material were first established not by Sherwood, but by the Russian agrophysicists P. S. Kossovich and A. F. Lebedev. It appears that at the beginning of the twentieth century, while investigating the evaporation of moisture from the soil, Kossovich established the existance of two periods in the evaporation process. In the second period Kossovich also noted the presence of a certain singular point. Essentially, he anticipated Sherwood's classification of the mechanism of the drying process. Professor Lebedev's research of about the same date showed that during the evaporation of moisture from the soil, evaporation takes place inside the soil with subsequent migration of moisture in vapor form. Thus, the depression of the evaporation zone was first noted, though very roughly, in the work of A. F. Lebedev.

The progress made during this decade is reviewed in the monograph "Theory of Drying," which was awarded a state prize in 1951.

The next decade 1950–1960 was characterized by the vigorous development of research in external heat and moisture transfer and heat and moisture transfer inside drying material. This research was generally based on the interrelationship of heat and moisture transfer.

Attention was concentrated on the mechanism of drying by high-frequency currents and infrared radiation. P. D. Lebedev established that high drying rates moisture transfer is determined not only by the moisture-content gradient and the temperature gradient but also by the total pressure gradient. This research was confirmed by the direct experiments of G. A. Maksimov, as a result of which the interrelationship between the differential equations of heat and moisture transfer became even more complicated and three differential equations were employed to describe the transfer process (Yu. A. Mikhailov).

It was found that the external heat and moisture transfer is continuously linked with the internal moisture and heat transfer. In particular, Lebedev established that during the period of falling drying rate the Nusselt number is a function of the moisture content.

A. V. Nesterenko and N. F. Dokuchaev found that in the evaporation of a liquid from a free surface the temperature field in the boundary layer is not similar to the vapor concentration field. These were important experimental results.

For a long time it was assumed, as in some countries it still is, that mass transfer is similar to heat transfer in drying processes, and therefore the temperature and vapor concentration fields in the boundary layer were assumed to be similar.

Soviet scientists have shown that no such similarity exists. Accordingly, the classical formulas of heat transfer and mass transfer cannot be used to describe the laws of heat and mass transfer in the drying process.

The heat and mass transfer associated with the evaporation of a liquid from a free surface or the evaporation of moisture from moist material is characterized by the Gukhman number. It is interesting to note that the introduction of the Gukhman number into equations relating the Nusselt, Reynolds, and Prandtl numbers generated lively debate. Only recently has the Gukhman number been granted full rights of citizenship.

Operational calculus and the methods of finite integral transforms were used to solve the system of differential equations of heat and mass transfer. Solutions of this system for various boundary conditions were systematized for publication in the monograph "Theory of Energy and Mass Transfer." This monograph has gone through two Russian editions and has been translated into English.

The same decade was notable for progress in the study of radiant-heat drying. Workers at the Moscow Power Engineering Institute established that at the beginning of drying there is a redistribution of moisture, in which the dominant factor is thermodiffusion.

It has been established that the evaporation zone is depressed into the material in accordance with a linear law. Research by the Physics Department of MTIPP has established that the partial vapor pressure is not the external moisture transfer potential, and that the relative humidity of the air can serve as a first approximation. This is in full agreement with later research which established that the chemical potential of moist air is proportional to the logarithm of the relative humidity. It has also been established that the temperature field and the vapor concentration field in the boundary layer are not similar, and that during the period of falling drying rate the heat-transfer and mass-transfer Nusselt numbers continuously decrease with time. Furthermore, the evaporation zone is depressed into the material in accordance with a linear law with a critical point corresponding to the critical moisture content of the material.

At the end of this period of development of the drying theory it was possible to assume that the system of differential transport equations and its solution could serve as a basis of studying the dynamics of drying processes.

Since 1960, in connection with the development of space technology, much attention has been given to so-called transpiration of mass-transfer cooling.

Research on heat and mass transfer in rheological systems has been conducted in connection with the development of the hydrodynamics of heat and mass transfer in non-Newtonian liquids. This research has had important applications in the chemical industry. Similarly, work is now being done on heat and mass transfer in relaxation-polarized systems, a more general case of rheological systems. This work is of great importance in connection with the study of heat and mass transfer in moist materials in the presence of a variable electromagnetic field.

For some time work on the influence of a variable magnetic field on heat and mass transfer was regarded with suspicion. However, this problem has now been rigorously formulated and solved and it has been shown that a variable magnetic field does affect the rate of chemical reactions as well as the heat and mass transfer.

Direct experiments on the kinetics of absorption of liquid elements by capillary and capillary-porous bodies have shown that an electromagnetic field has an important influence on the kinetics of the process.

In our view, there are good prospects for using inhomogeneous and variable magnetic fields to modify structural and mechanical properties and the kinetics of heat and mass transfer, as well as the rates of chemical reactions.

Similarly, acoustical methods can be used to intensify the drying process and directly influence heat and moisture transfer in moist materials.

Thus, the analytical theory of heat and moisture transfer is now essentially part of a transport theory derived from the basic principles of the thermodynamics of irreversible processes (Yu. A. Mikhailov and A. G. Temkin).

Work on the potential theory of moisture transport deserves particular attention. This theory was first advanced in the early fifties in connection with research in thermophysics. The essence of this theory is the idea that the transport potential is not the moisture content but a certain thermodynamic quantity that can be determined experimentally.

By analogy with the heat-temperature transfer potential, a moisture transfer potential was introduced, and a method of experimentally determining the potential from the moisture content of a standard body was developed. This research has since been generalized and its relationship with the theory of chemical potentials has been established.

Thus, the potential theory of moisture transport has now been given a sound basis. It is of great practical significance in connection with the development of new drying techniques based on contact moisture transfer. Drying systems based on the potential theory of contact moisture transfer have proved themselves in practice.

For a long time the principal obstacle to the application of mathematical transport theory of engineering calculations was the lack of experimental data on the transport coefficients. A large amount of material on the thermophysical characteristics of moist materials has now been accumulated and a fairly simple relationship between the diffusion coefficient, moisture content, and temperature, valid for most moist materials, has been established.

Moreover, methods have been developed for solving the nonlinear system of differential transport equations on electronic computers (programs for various types of computers have been published). This will undoubtedly serve to accelerate the introduction of analytical methods into engineering practice.

Of the numerous empirical formulas proposed for calculating the drying rate, that most commonly used is the simple formula according to which the drying rate is directly proportional to the moisture content removed. The proportionality factor, called the drying coefficient, can be calculated from a rather simple relation: the relative drying coefficient is equal to 1.8 divided by the initial moisture content.

On the basis of the heat-balance equation a direct relation has been established between the rate of heat transfer, the rate of moisture transfer, and the heating rate of the moist material during the drying process. By introducing the so-called Rebinder number it has proved possible to establish a direct relation between the average moisture content and the average temperature of the material.

Direct experiments have established that the Rebinder number does not depend on the velocity in air of the moisture content and is a very general characteristic of the kinetics of drying processes. The Rebinder number can be expressed in terms of the so-called dimensionless temperature coefficient of drying and the Kossovich number. As a result a very simple relation is obtained: the kinetics of the drying process is determined by two coefficients—the drying coefficient and the temperature coefficient of drying.

Drying kinetics, as distinct from drying dynamics, is understood as the relation between the mean integral characteristics of the body (drying curves, temperature curves).

The introduction of these new criteria made it possible to close the gap between drying theory and practical engineering calculations.

However, it cannot be said that all the problems of drying theory have now been solved. Many uninvestigated questions remain. The mathematical theory of drying must be developed with machine techniques. It must be regarded as part of the general problem of heat and mass transfer in systems with an erasable memory.

It is necessary to continue experimental work on the determination of the thermophysical characteristics of moist materials. Research on heat and moisture transfer under variable drying conditions is particularly promising and important.

As already noted, the intensification of drying processes and the heat treatment of materials using a variable electromagnetic field are developing areas of particular interest. A number of new ideas have been proposed in connection with heat and mass transfer at low temperatures in a deep vacuum; in particular, some very interesting work is being done in connection with various problems of molecular drying.

5. SOME TOPICAL PROBLEMS OF HEAT AND MASS TRANSFER

Heat conduction. In the mathematical theory of heat conduction one of the most pressing problems is to develop methods of solving nonstationary nonlinear problems of heat conduction when the thermophysical characteristics are functions of temperature.

In the area of experimental methods of determining thermophysical characteristics, an important question is the use of machine techniques to analyze data on the nonstationary temperature field, with the object of obtaining numerical data on all the transport coefficients from a single experiment.

A large amount of material on the thermophysical characteristics of homogeneous and isotropic bodies has already been accumulated; accordingly, the next task is to develop methods of calculating transport coefficients from given structural characteristics with allowance for multiphase bound matter. However, it is still necessary to accumulate experimental data on the effective (over-all) thermophysical characteristics of materials which experience phase transitions and chemical reactions in the working temperature interval. This work is important in connection with the development of new materials (plastics, polymer compounds, etc.).

The analytical theory of the interrelated system of differential equations of energy and mass transfer, following from the Onsager system of equations, also requires development.

Heat transfer in a single-phase medium. The system of Navier-Stokes equations for a laminar boundary layer has now been completely programmed and numerical solutions have been obtained on electronic computers over a broad interval of basic transport characteristics. The numerical solutions make it possible to take into account changes in the thermophysical characteristics, the effect of blowing and suction in the boundary layer, the effect of thermodiffusion, etc. The results obtained have been analyzed in some detail.

The principal problem of the theory of convective heat transfer is the correct physical formulation of the boundary conditions.

The fact is that heat transfer between a fluid flow and a surface is usually analyzed for given and constant surface conditions (e.g., temperature).

However, in many practical applications (high-intensity heat transfer, heat sources in the body, etc.) the surface temperature is not constant and cannot be preassigned as a function of the coordinate of the surface, since it is determined by the heat-transfer process itself and depends on the interaction between the flow and the body. Neglecting this may lead to serious errors in calculating the heat transfer. Obviously, in the general case the surface conditions should be determined from a joint solution of the equations of heat transfer in the body and the flow, together with the equations of hydrodynamics.

The temperature of the body-fluid interface and the flow through that interface, determined from the solution of the problem, make it possible to calculate the heat-transfer coefficient.

It is easy to suggest examples of practical importance in which the local heat-transfer coefficient α becomes infinite or negative at particular points on the surface. Therefore the ordinary determination of the coefficient is essentially unsuitable for flows with variable surface temperature. From the physical standpoint, this unsuitability of the heat-transfer coefficient for describing heat transfer with variable surface temperature is due to the fact that it is determined only from the value of the surface temperature at a given point, whereas the properties of the flow at some point depend not only on the temperature at that point but also on the entire upstream temperature distribution along the surface.

The physical formulation of conjugate heat-transfer problems was first considered by T. L. Perel'man who proposed methods of asymptotic solution. A. V. Luikov and co-workers have examined the problem of approximating the boundary conditions at the body-flow interface by constructing differential operators relating the temperature of the interface and the flow of heat through it.

The widespread utilization of radiative heat exchange and high heat flux densities make the investigation of such problems a matter of urgency. One of the principal problems in this area is the development of engineering methods of calculation using digital and analog computers and the development of experimental methods of determining the opticogeometric characteristics of radiative heat exchange.

The vigorous development of the chemical industry and the use of new methods of hydrothermal material processing have drawn attention to the hydrodynamics and heat transfer of non-Newtonian liquids, i.e., to the motion and transfer of heat in rheological systems.

The solution of this problem is not only of great practical importance, it is also of purely theoretical interest for the further development of the theory of transport in continua.

Another important problem is the investigation of nonstationary convective heat transfer. As mathematical and experimental studies have shown, the laws of nonstationary convective heat transfer differ substantially from the laws of stationary convective heat transfer, some of the latter being essentially inapplicable to high-intensity nonstationary heat-transfer processes.

The development of this branch of research is of considerable practical importance.

The central problem of conductive heat transfer is the problem of turbulent heat and mass transfer. Despite numerous studies of a mathematical and experimental nature, this problem still awaits final solution. Obviously, new ideas are required. The theoretical and practical significance of this problem is obvious. In many processes, convective heat transfer takes place simultaneously with radiative heat exchange. Hence the interaction of these processes needs to be investigated.

Heat and mass transfer with phase transitions and chemical reactions. Heat and mass transfer in multicomponent systems in the presence of phase transitions and chemical reactions is of great interest. It is common in chemical engineering processes. However, it has so far received little attention. This is because of the complexity of the transfer phenomena and the presence of secondary effects associated with the chemical reactions.

The mathematical description of the transfer processes under these conditions takes the form of a complex system of differential equations whose solution is practically impossible. One means of obtaining an engineering solution of this problem is the method of description and calculation proposed by Professor Spalding in his treatise "Convective mass transfer." This method gives a sufficiently accurate and uniform description of heat and mass transfer in multicomponent systems in the presence of phase transitions and chemical reactions. Thermodynamic relations are widely used for calculating the transfer potentials. It is important to note that diverse transport phenomena encountered in chemical engineering processes are described by common laws. This method is very promising for the engineering solution of the problem of the kinetics of chemical-engineering processes.

Heat and mass transfer at high speeds and temperatures. Heat and mass transfer at supersonic speeds is of great importance in modern technology. Usually the problem is solved for flight in the dense layers of the atmosphere. Heat and mass transfer is complicated by the presence of dissociation, chemical reactions, and a number of other effects.

The study of heat and mass transfer in a low-temperature plasma is of considerable interest in connection with chemical-engineering processes. Plasmochemistry is a new branch of science of more than purely theoretical interest. There are plasma jet processes for obtaining bound nitrogen from atmospheric air, industrial hydrogen from natural gas, and a number of other chemical important substances. Moreover, low-temperature plasmas are also used in metallurgy. All this has created an urgent need for research into heat and mass transfer in subsonic and supersonic plasma flows. This is a very complicated problem. For example, when a body is placed in a plasma flow, surface charges may develop when it is neutral. In this case the ordinary equations of aerodynamics are invalid and it is necessary to introduce an equation of conservation of charge density. The form of this equation is oscillatory in character, the fundamental frequency depending on the concentration of charge carriers creating the excess plasma charge.

The experimental determination of the transport coefficients and thermodynamic characteristics of low-temperature plasma is a very topical problem.

However, apart from these fundamental problems there are numerous special problems of the theory of heat and mass transfer that require solution.

We have considered only the latest and most pressing problems in this area.

It is to be hoped that this review, in spite of its brevity and incompleteness, will give some idea of the principal achievements of Soviet science in the field of heat and mass transfer over the last 50 years.

The Editors